

Chapter 8: The Binomial & Geometric Distributions

8.1 The Binomial Distributions - Day 1

Obs. you will find the number of successes in a Fixed number of trials.

The Binomial Setting - Bottom of Pg 439

- 1) Each obs. falls into just two categories, which for convenience we call "success" or "failure".
- 2) There is a fixed # n of observations.
- 3) The n observations are all independent.
- 4) The prob. of success, call it p , is the same for each obs.

* $\text{pop} \geq 10n$

The Binomial Setting occurs when you want to know the # of successes in a fixed # of trials.

The Binomial Random Variable is X (# of successes) if Data are produced in the Binomial Setting.

The Binomial Distribution is the distribution of the count X of successes w/parameters n and p . (n - # of obs., p - prob. of success of any one obs., Possible values of X are the whole #'s from 0 to n)

Notation: $X = B(n, p)$

Example 8.1

Example 8.2

Example 8.3 $\rightarrow$ + P after ($\text{Pop.} \geq 10n$)

8.1

Example 8.5 (Also see cdf on pg. 457)

Show binompdf (10, .1)

pdf - Probability Distribution Function - Assigns a prob. to x
 $\text{binompdf}(n, p, x)$

Finish Ex 8.5 $\text{binompdf}(10, .1, 0) + \text{binompdf}(10, .1, 1) = .7361$

Histogram of PDF (Skew R $\rightarrow p < .5$, skew L $\rightarrow p > .5 \Rightarrow$)

cdf - cumulative Distribution Function - calculates probabilities that are $\leq x$.

$\text{binomcdf}(n, p, x)$

$$P(0) + P(1) + P(2) + \dots + P(x)$$

Histogram of CDF - see pg 457 (Always skewed L)

Now, Redo 8.5 w/ binomialcdf notation $\rightarrow \text{binomcdf}(10, 0.1)$
E.73

8.5

8.7

HW:

8.3, 8.4, 8.8, 8.60 now

8.10-13 (optional)

8.1 The Binomial Distribution - Day 2

QBS You will evaluate probabilities using notation for Binomial Settings.

Binomial Coefficient - The # of ways of arranging k successes among n observations.

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} \quad \left. \vphantom{\frac{n!}{k!(n-k)!}} \right\} \text{ Same as } nCr \text{ in Alg II / Trig. (Pre Calc)}$$

Binomial Probability - $P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$
 $= \binom{n}{k} p^k q^{n-k}$

We show this:

$$nCr p^r q^{n-r}$$

n = # of trials

k = # of "successes" (or r)

p = Prob. of "success"

$1-p = q$ = Prob. of "Failure"

8.10

8.11

8.12

8.13

8.60 - pg 450

DW: 8.10 - 8.13

8.1 The Binomial Distribution - Day 3

OBJ You will find the mean + st. Dev of a rand. var.

If a basketball player makes 75% of her free throws, what would the mean (μ) be in 20 attempts?

$$75\% \text{ of } 20 = .75 \times 20 = 15 \text{ so...}$$

$$\begin{aligned} \mu &= np \\ \sigma &= \sqrt{np(1-p)} \end{aligned} \quad \left. \begin{array}{l} \text{+ on P.S.} \\ \text{Binom.} \\ \text{Random} \\ \text{Variable} \end{array} \right\}$$

Proof of Both are on pg 450 (if interested)

(Notation for a normal Approx. for a Binomial Dist:

$\rightarrow N(np, \sqrt{np(1-p)}) \rightarrow$ Use this when finding the prob. w/ a z -score
 CONDITIONS: $np \geq 10$
 $n(1-p) \geq 10$ } This ensures a large sample size.

* As n increases, the NORMAL APPROX. Improves (you know more about your population).

* Most accurate when $p \approx \frac{1}{2}$

* Least accurate when $p \approx 0$ or $p \approx 1$

AP Calc

$$x + y = 1$$

$$y = 1 - x$$

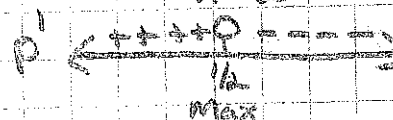
$$p = xy$$

$$p = x(1-x)$$

$$p = x - x^2$$

$$p' = 1 - 2x = 0$$

$$x = \frac{1}{2}$$



8.16

8.20

show $\text{binompdf}(5, .65)$ gives all 6 probabilities.

HW: 8.17, 8.28, 8.30, 8.34

8.2 The Geometric Distributions - Day 1

OBS: You will determine how many trials it will take to observe a particular event.

Binomial - n is fixed beforehand.

Geometric - Looking for n until you observe the event.

- Possible values x or n : 1, 2, 3, ..., b/c it is theoretically possible to proceed indefinitely w/o obtaining a success. (Flip a coin until H, Roll a die until 4, shoot a foul court basket until you make it).

The Geometric Setting - Bottom of pg. 464

- 1) Each obs. falls into one of two categories "success" or "failure."
- 2) The prob. of success, p , is the same for all obs.
- 3) The obs. are all independent.
- 4) The variable of interest is the number of trials required to obtain the first "success."

Example 8.15

Example 8.16

Roll a Die until you get a 4.

Success on Roll #	X	$P(X=)$
1	1	$1/6$
2	2	$5/6 \cdot 1/6$
3	3	$5/6 \cdot 5/6 \cdot 1/6$
4	4	$5/6 \cdot 5/6 \cdot 5/6 \cdot 1/6$
...	...	...
100	100	$(5/6)^{99} (1/6)$
...	...	...
n	n	$(5/6)^{n-1} (1/6)$

Top of pg 466

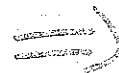
$$P(X=n) = (1-p)^{n-1} (p)$$

TI-84

1/6 ENTER
x 5/6 ENTER
ENTER
ENTER
ENTER

Figure 8.4 (Pg 467) Reflects these Probabilities

geom Dist Hist. are Always skewed R+ b/c you are wait. the previous prob. are $0 \leq 1$.



Sum of a geometric Series:

$$S_n = \frac{a_1 - a_1 r^n}{1 - r}$$

a_1 = 1st term

r = common ratio: $(1-p)$ or q or prob of "Failure"

n = # of terms

$$S_n = \frac{a_1 (1 - r^n)}{1 - r}$$

$$\text{As } n \rightarrow \infty$$

$$r^n \rightarrow 0$$

$S_n \dots$

$$S_n = \frac{a_1 (1 - 0)}{1 - r}$$

$$S_n = \frac{a_1}{1 - r} \quad \text{As } n \rightarrow \infty$$

geompdf (p, x)

geomcdf (p, x)

$$\downarrow$$
$$P(0) + P(1) + P(2) + \dots + P(x)$$

8.38

8.37

8.39

HW: Finish (8.37-8.39) If not done in class.

8.2 The Geometric Distributions - Day 2

OBJ: You will use the mean & st.d, Dev of a geometric Random variable.

Flip a coin until you get a Head:

X	1	2	3	4	...
P(X)	p	pq	pq ²	pq ³	...

$$\mu_x = \frac{1}{p}$$

$$\sigma_x^2 = \frac{1-p}{p^2}$$

Variance

Proofs on pg 434

$$\sigma_x = \sqrt{\frac{1-p}{p^2}}$$

St. Dev

The probability that it takes more than n trials to see the first success is:

Proof on pg 434

$$P(X > n) = (1-p)^n \quad \text{or} \quad 1 - \text{geomcdf}(p, n)$$

I Like This Better !!

Example 8.19

8.43

8.44

HW: 45, 47 a, b } geometric
56, 57 } binomial

Go over
#63 (a-d)
w/ HW