Chapter 8: The Binomial & Geometric Distributions
8.1 The Binomial Distributions - Day 1

OBS You will find the number of successes in a fixed number of trials.

The Binomial Setting - Bottom of Pg 439

1) Each obs. falls into just two categories, which for convenience we call "success" or "failure".
2) There is a fixed # n of observations.
3) The n observations are all independent.
4) The prob. of success, call it p, is the same for each obs.
   * pop $\geq$ 10n

The Binomial Setting occurs when you want to know the # of successes in a fixed # of trials.

The Binomial Random Variable is X(# of successes) if Data are produced in the Binomial Setting.

The Binomial Distribution is the Distribution of the count X of successes w/parameters n and p. (n - # of OBS., p - prob. of success of any one OBS. Possible values of X are the whole #'s from 0 to n)

Notation: X = B(n, p)

Example 8.1
Example 8.2
Example 8.3 → + TP after (Pop. $\geq$ 10n)

8.1

Example 8.5 (Also see cdf on pg. 457)

Show binompdf (10, .1)

pdf - Probability Distribution Func - Assigns a prob. to x binompdf (n, p, x)

Finish Ex 8.5 binompdf (10, .1, 0) + binompdf (10, .1, 1) = .7361

Histogram of PDF (Skew R → p < .5, Skew L → p > .5 ⇒)

cdf – Cumulative Distribution Function – Calculates probabilities that are $\leq x$,

binomcdf (n, p, x)

$P(0) + P(1) + P(2) + \ldots + P(X)$

Histogram of CDF – See pg 457 (Always Skewed L)

Now, Redo 8.5 w/ binomialcdf Notation → binomcdf $(10, .1, 1)$ $\boxed{= .73}$

8.5

8.7

HW: 8.3, 8.4, 8.8, $\boxed{8.60}$ new

8.10–13 (Optional)

8.1 The Binomial Distribution – Day 2

OBJ You will evaluate probabilities using notation for Binomial Settings.

Binomial Coefficient – The # of ways of arranging K successes among n observations.

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$ } Same as $_nC_r$ in Alg II / Trig. (Pre Calc)

Binomial Probability – $P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$

$$= \binom{n}{k} p^k q^{n-k}$$

We show this:

$$_nC_r\, p^r q^{n-r}$$

n = # of trials
k = # of "successes" (or r)
p = Prob. of "success"
$1-p = q$ = Prob. of "Failure"

8.10
8.11
8.12
8.13

8.60 – pg 460

HW: 8.10 – 8.13

8.1 The Binomial Distribution - Day 3

OBJ. You will find the mean + st. Dev of a rand. var.

If a basketball player makes 75% of her free throws, what would the mean (μ) be in 20 attempts?

75% of 20 = .75 × 20 = 15 so.... $\boxed{\begin{matrix}\mu = np \\ \sigma = \sqrt{np(1-p)}\end{matrix}}$ for on F.S. } Binom. Random Variable

Proof of Both are on pg 450 (if interested)

Notation for a normal Approx. for a Binomial Dist:

→ $N\left(np, \sqrt{np(1-p)}\right)$ → Use this when finding the prob. w/ a z-score

CONDITIONS: $np \geq 10$, $n(1-p) \geq 10$ } This ensures a large sample size.

* As n increases, the NORMAL APPROX. Improves (you know more about your population).

* Most accurate when $p \approx 1/2$

* Least accurate when $p \approx 0$ or $p \approx 1$

} $x+y=1$, $y=1-x$

AP Calc

$p = xy$

$p = x(1-x)$

$p = x - x^2$

$p' = 1 - 2x = 0$

$x = \frac{1}{2}$

p' ← + + + + 0 − − − − →, $1/2$ Max

8.16

8.20

show binompdf (5, .65) gives all 6 probabilities.

HW: 8.17, 8.28, 8.30, 8.34

8.2 The Geometric Distributions – Day 1

<u>OBJ</u>: You will determine how many trials it will take to observe a particular event.

Binomial – n is fixed beforehand.
Geometric – Looking for n until you observe the event.
- Possible values $X = n$: 1, 2, 3, ..., b/c it is theoretically possible to proceed indefinitely w/o obtaining a success. (Flip a coin until H, Roll a die until 4, shoot a full court basket until you make it).

<u>The Geometric Setting</u> – Bottom of pg. 464

1) Each obs. falls into one of two categories "success" or "failure."
2) The prob of success, p, is the same for all obs.
3) The obs. are all independent.
4) The variable of interest is the number of trials required to obtain the first "success."

Example 8.15
Example 8.16

Roll a Die until you get a 4.

Success on Roll #	X	P(X=)
1	1	$1/6$
2	2	$5/6 \cdot 1/6$
3	3	$5/6 \cdot 5/6 \cdot 1/6$
4	4	$5/6 \cdot 5/6 \cdot 5/6 \cdot 1/6$
⋮	⋮	⋮
100	100	$(5/6)^{99}(1/6)^1$
⋮	⋮	⋮
n	n	$(5/6)^{n-1}(1/6)$

Top of pg 466

$$P(X=n) = (1-p)^{n-1}(p)$$

<u>TI-84</u>
1/6 $\boxed{\text{ENTER}}$
× 5/6 $\boxed{\text{ENTER}}$
$\boxed{\text{ENTER}}$
$\boxed{\text{ENTER}}$
⋮

Figure 8.4 (Pg 467) Reflects these Probabilities

Geom Dist Hist. are <u>Always</u> Skewed Rt b/c you are mult. the previous prob. by a # < 1. ⟹

Sum of a Geometric Series:

$S_n = \frac{a_1 - a_1 r^n}{1-r}$

a_1 = 1st term
r = common ratio: $(1-p)$ or q or prob of "Failure"
n = # of terms

$1/6$
$5/6 \quad 1/6$
$5/6 \quad 5/6 \quad 1/6$
$\underbrace{5/6 \quad 5/6 \quad 5/6 \quad 1/6}$

$\therefore r = 5/6 = (1-p)$

$S_n = \frac{a_1(1-r^n)}{1-r}$

As $n \to \infty$
$r^n \to 0$

So......

$S_n = \frac{a_1(1-0)}{1-r}$

$\boxed{S_n = \frac{a_1}{1-r} \quad \text{As } n \to \infty}$

geompdf (p, x)

geomcdf (p, x)

$\downarrow$

$P(0) + P(1) + P(2) + \dots + P(x)$

8.38
8.37
8.39

<u>HW</u>: Finish (8.37 - 8.39) If not done in class.

8.2 The Geometric Distributions – Day 2

OBJ: You will use the mean & St. Dev of a Geometric Random Variable.

Flip a coin until you get a Head:

X	1	2	3	4 ....
P(X)	p	pq	pq^2	pq^3 ....

$$\boxed{\mu_x = \frac{1}{p} \qquad \underbrace{\sigma_x^2 = \frac{1-p}{p^2}}_{\text{Variance}}}$$

Proofs on pg 480

$$\underbrace{\sigma_x = \sqrt{\frac{1-p}{p^2}}}_{\text{St. Dev}}$$

The probability that it takes <u>more</u> <u>than</u> n trials to see the first success is:

Proof on pg 490

$$\boxed{p(x > n) = (1-p)^n} \quad \text{or} \quad \underbrace{1 - \text{geomcdf}(p, n)}_{\text{I Like This Better !!}}$$

:)

Example 8.19

8.43
8.49

HW: 45, 47 a, b } Geometric
56, 57 } binomial

Go over #63 (a-d) w/ HW